

ALGEBRAIC GENERAL TOPOLOGY (THEORY OF FUNCODS AND RELOIDS)

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In my book [1] I introduce some new concepts generalizing general topology, including *funcoids* and *reloids*. The book along with supplementary materials (such as a partial draft of the second volume) is freely available (including the L^AT_EX source) online.

Before studying funcoids, in my book I consider co-brouwerian lattices and lattices of filters in particular, as the theory of funcoids is based on theory of filters. My book contains probably the best (and most detailed) published overview of properties of filters.

Because you always can refer to my book, in this short intro I present theorems without proofs.

I denote order on a poset as \sqsubseteq and corresponding lattice operations as \sqcup and \sqcap . I denote the least and greatest elements (if they exist) of our poset as \perp and \top correspondingly.

1. FILTERS

Definition 2. I order the set \mathfrak{F} of filters (including the improper filter) *reverse* to set-theoretic order, that is $\mathcal{A} \sqsubseteq \mathcal{B} \Leftrightarrow \mathcal{A} \supseteq \mathcal{B}$ for $\mathcal{A}, \mathcal{B} \in \mathfrak{F}$.

Proposition 3. *This makes the set of filters on a set into a co-brouwerian (and thus distributive) lattice, that is we have $\mathcal{A} \sqcup \sqcap S = \sqcap_{\mathcal{X} \in S} (\mathcal{A} \sqcup \mathcal{X})$ for a set S of filters and a filter \mathcal{A} .*

4. FUNCODS

Let $\mathfrak{F}(A), \mathfrak{F}(B)$ be sets of filters on sets A, B . They are complete atomistic co-brouwerian lattices.

Definition 5. A *funcoid* $A \rightarrow B$ is a quadruple (A, B, α, β) where α and β are functions $\mathfrak{F}(A) \rightarrow \mathfrak{F}(B)$ and $\mathfrak{F}(B) \rightarrow \mathfrak{F}(A)$ correspondingly, such that $\mathcal{Y} \sqcap \alpha(\mathcal{X}) \neq \perp \Leftrightarrow \mathcal{X} \sqcap \beta(\mathcal{Y}) \neq \perp$ for every $\mathcal{X} \in \mathfrak{F}(A), \mathcal{Y} \in \mathfrak{F}(B)$.

Definition 6. I denote $(A, B, \alpha, \beta)^{-1} = (B, A, \beta, \alpha)$.

Definition 7. I denote $\langle (A, B, \alpha, \beta) \rangle = \alpha$.

Funcoids generalize such things as:

- binary relations;
- proximity spaces;
- pretopologies;
- preclosures.

For a proximity δ , define

$$\mathcal{X} \delta' \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y} : X \delta Y$$

for all filters \mathcal{X}, \mathcal{Y} . Then we have a unique funcoid f such that

$$\mathcal{X} \delta' \mathcal{Y} \Leftrightarrow \mathcal{Y} \sqcap \langle f \rangle \mathcal{X} \neq \perp \Leftrightarrow \mathcal{X} \sqcap \langle f^{-1} \rangle \mathcal{Y} \neq \perp.$$

Definition 8. $\mathcal{X} [f] \mathcal{Y} \Leftrightarrow \mathcal{Y} \sqcap \langle f \rangle \mathcal{X} \neq \perp \Leftrightarrow \mathcal{X} \sqcap \langle f^{-1} \rangle \mathcal{Y} \neq \perp$.

Proposition 9. *A funcoid $f : A \rightarrow B$ is uniquely determined by $\langle f \rangle$ and moreover is uniquely determined by values of the function $\langle f \rangle$ on principal filters or by the relation $[f]$ between principal filters.*

See my book for formulas for *principal funcoids* that is funcoids corresponding to binary relations and for funcoids corresponding to pretopologies and preclosures (particularly funcoids corresponding to topological spaces, as topological spaces can be considered as a special case of either pretopologies or preclosures).

There are also several other equivalent ways to define funcoids.

Funcoids are made more interesting than topological spaces by a new operation (missing in traditional general topology), *composition*, which is defined by the formula

$$(B, C, \alpha_2, \beta_2) \circ (A, B, \alpha_1, \beta_1) = (A, C, \alpha_2 \circ \alpha_1, \beta_1 \circ \beta_2).$$

10. RELOIDS

Reloids are basically just filters on cartesian product $A \times B$ of two given sets A and B .

Formally:

Definition 11. A *reloid* is a triple (A, B, F) where A, B are sets and F is a filter on $A \times B$.

Reloids are a generalization of uniform spaces and of binary relations.

Definition 12. Composition $g \circ f$ of reloids can be easily defined as the reloid determined by the filter base consisting of compositions of binary relations defining these reloids (see the book for an exact formula).

Proposition 13. *Composition of reloids (and of funcoids) is associative.*

The sets of funcoids and reloids constitute complete atomistic co-brouwerian lattices and these lattices have interesting properties.

There are interesting relationships between funcoids and reloids, as well as special classes of funcoids and reloids.

14. CONTINUITY

A function f from a space μ to a space ν is *continuous* iff $f \circ \mu \sqsubseteq \nu \circ f$. This formula works for continuity, proximal continuity, uniform continuity, etc., so making all kinds of continuity described by the same formula.

15. OTHER

My book [1] also considers pointfree generalizations of funcoids, multidimensional generalizations of funcoids and reloids and other research topics.

I also introduce generalized limit for arbitrary (not necessarily continuous) functions.

My work introduced *many* new conjectures. So I give you a work.

REREFENCES

- [1] Victor Porton. Algebraic General Topology. Volume 1.
<http://www.mathematics21.org/algebraic-general-topology.html>